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ON THE RANGE KERNEL ORTHOGONALITY AND P–SYMMETRIC OPERATORS

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Abstract. Let H be a separable infinite dimensional complex Hilbert space, and let $L(H)$ denote the algebra of all bounded linear operators on H. For given $A \in L(H)$, we define the derivation $\delta_A : L(H) \longrightarrow L(H)$ by $\delta_A(X) = AX - XA$. In this paper we establish the orthogonality of the range $R(\delta_A)$ and the kernel $\ker(\delta_A)$ of a derivation δ_A induced by a cyclic subnormal operator A, in the usual sense. We give a version of the Putnam - Fuglede theorem. We establish a short proof of the principal result of F. Wenying and J. Guoxing in [10]. Relatad results for P-symmetric operators are also given.

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