

SOME COMPARISON INEQUALITIES FOR GINI AND STOLARSKY MEANS

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Abstract. We study the following inequality

$$G_{a,b}(x,y) \leq S_{c,d}(x,y) \quad (x,y \in \mathbb{R}_+),$$

where $G_{a,b}(x,y)$ stands for the so-called Gini mean of the positive variables x and y , and $S_{c,d}(x,y)$ is the Stolarsky mean of them. We give some necessary conditions for the parameters a, b, c, d , then, distinguishing the different positions of the Gini-parameters, we present some sufficient conditions for the inequality above to hold for variables indicated. In some special cases it turns out that these conditions coincide.

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