

PHRAGMÉN–LINDELÖF TYPE ALTERNATIVE RESULTS FOR THE STOKES FLOW EQUATION

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Abstract. In this paper, we derive estimates for weighted energy expression for the solution of the Stokes flow equation in a semi-infinite plane channel by means of a second order differential inequality. From the estimates, we establish Phragmén-Lindelöf alternative that the solutions either grow or decay exponentially. In the case of decay, we also show how to bound the total weighted energy.

Mathematics subject classification (2000): 35B40, 35K50, 35K45.

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