

## MONOTONICITY OF RATIO BETWEEN THE GENERALIZED LOGARITHMIC MEANS

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*Abstract.* Let  $c > b > a > 0$  be real numbers. Then the function  $f(r) = \frac{L_r(a,b)}{L_r(a,c)}$  is strictly decreasing on  $(-\infty, \infty)$ , where  $L_r(a, b)$  denotes the generalized (extended) logarithmic mean of two positive numbers  $a$  and  $b$ .

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