

ON BEHAVIOUR OF THE RIESZ AND GENERALIZED RIESZ POTENTIALS AS ORDER TENDS TO ZERO

A. D. GADJIEV, A. ARAL AND ILHAM A. ALIEV

Abstract. In this paper, we present the Riesz potentials I^α and the generalized Riesz potentials I_V^α as the families of positive linear operators, depending on parameter $\alpha > 0$. We investigate their pointwise convergence and convergence in the norm as $\alpha \rightarrow 0$. We investigate also the order of approximation of these families and show in particular that the order of approximation at the Lipschitz points is independent from Lipschitz degree.

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