

WEAK NONCOMPACTNESS IN BANACH SEQUENCE SPACES AND ITS EXTRAPOLATION PROPERTIES

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Abstract. Explicit formulae in selected Banach sequence spaces are established for the measure of weak noncompactness based on James' criteria. Estimates of the deviation from weak compactness are given for bounded linear operators extrapolated by the Jawerth-Milman Σ_p and Δ_p methods for $1 < p < \infty$.

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