

A NEW SYSTEM OF GENERALIZED NONLINEAR MIXED VARIATIONAL INEQUALITIES

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Abstract. In this paper, we introduce and study a new system of generalized nonlinear mixed variational inequalities which contains some classes of variational inequalities and systems of variational inequalities in the literature as special cases. We prove the existence and uniqueness of solution and the convergence of some new n -step iterative algorithms with mixed errors for this system of generalized nonlinear mixed variational inequalities and its special cases. The results in this paper unify, extend and improve some known results in the literature.

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