Mathematical Inequalities & Applications
Volume 11, Number 3 (2008), 549–557

STABILITY OF LINEAR MAPPINGS IN QUASI–BANACH MODULES

MOHAMMAD SAL MOSLEHIAN AND GHADIR SADEGHI

Abstract. A quasi norm is a non-negative function $\|.\|$ on a linear space $\mathscr{X}$ satisfying the same axioms as a norm except for the triangle inequality, which is replaced by the weaker condition that "there is a constant $K \geqslant 1$ such that $\|x+y\| \leqslant K(\|x\|+\|y\|)$ for all $x,y \in \mathscr{X}$". In this paper, we prove the Hyers–Ulam–Rassias stability of linear mappings in quasi-Banach modules associated to the Cauchy functional equation and a generalized Jensen functional equation.

Mathematics subject classification (2000): 39B82, 39B52, 46L05, 47Jxx.
Key words and phrases: Cauchy equation, generalized Jensen equation, quasi-Banach module, C^*-algebra, Hyers–Ulam–Rassias stability, p-Banach module.

REFERENCES

[1] M. AMYARI, F. RAHBARNIA AND GH. SADEGHI, *Some results on stability of extended derivations*, J. Math. Anal. Appl. **329** (2007), no. 2, 753–758.
[2] Y. BENYAMINI AND J. LINDENSTRAUSS, *Geometric Nonlinear Functional Analysis, Vol. 1*, Colloq. Publ. **48**, Amer. Math. Soc., Providence, 2000.
[3] S. CZERWIK, *Functional Equations and Inequalities in Several Variables*, World Scientific, River Edge, NJ, 2002.
[4] Z. GAJDA, *On stability of additive mappings*, Internat. J. Math. Math. Sci. **14** (1991), 431–434.
[5] P. GĂVRUTA, *A generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. **184** (1994), 431–436.
[6] D. H. HYERS, *On the stability of the linear functional equation*, Pro. Nat'l. Acad. Sci. U.S.A. **27** (1941), 222–224.
[7] D. H. HYERS, G. ISAC AND TH. M. RASSIAS, *Stability of Functional Equations in Several Variables*, Birkhäuser, Basel, 1998.
[8] S.-M. JUNG, *Hyers–Ulam–Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press lnc. Palm Harbor, Florida, 2001.
[9] R. V. KADISON AND G. K. PEDERSEN, *Means and convex combinations of unitary operators*, Math. Scan. **57** (1985), 249–266.
[10] M. S. MOSLEHIAN, *Approximately vanishing of topological cohomology groups*, J. Math. Anal. Appl. **318** (2006), no. 2, 758–771.
[11] C. PARK, *On the stability of the linear mapping in Banach modules*, J. Math. Anal. Appl. **275** (2002), 711–720.
[12] C. PARK AND W. PARK, *On the Jensen's equation in Banach modules*, Taiwanese J. Math. **6** (2002), 523–531.
[13] TH. M. RASSIAS, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300.
[14] TH. M. RASSIAS AND P. ŠEMRL, *On the behavior of mappings which do not satisfy Hyers–Ulam stability*, Proc. Amer. Math. Soc **114** (1992), 989–993.
[15] TH. M. RASSIAS, *Functional Equations, Inequalities and Applications*, Kluwer Academic Publishers, Dordrecht, Boston and London, 2003.
[16] S. ROLEWICZ, *Metric Linear Spaces*, PWN-Polish Sci. Publ., Reidel and Dordrecht, 1984.
[17] J. TABOR, *Stability of the Cauchy functional equation in quasi-Banach spaces*, Ann. Polon. Math. **83** (2004), no. 3, 243–255.
[18] S. M. ULAM, *Problems in Modern Mathematics*, Wiley, New York, 1960.

Paper MIA-11-44