

ON GRONWALL–BELLMAN–BIHARI TYPE INTEGRAL INEQUALITIES IN SEVERAL VARIABLES WITH RETARDATION FOR DISCONTINUOUS FUNCTIONS

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Abstract. In this work some new nonlinear inequalities of the Gronwall-Bellman-Bihari type in n -independent variables with delay for discontinuous functions are presented. They have non-Lipschitz type discontinuities in some set of points for a n -dimensional Euclidean space. The results extend recent results of R. Bellman, I. Bihari, B. K. Bondge, V. Lakshmikantham, B. G. Pachpatte for continuous case of integral inequalities and its generalization for integro-sum inequalities (discontinuous case) presented in the investigations of D. Bainov, S. Borysenko, V. Lakshmikantham, S. Leela, A. Samoilenko.

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