

UNIFORM NON-SQUARENESS, UNIFORM NORMAL STRUCTURE AND GAO'S CONSTANTS

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Abstract. We consider two geometric constants, $f(X)$ and $E(X)$, introduced by Gao recently. An estimate concerning the James and Gao's constants is obtained. This estimate therefore allow us to get: (1) A Banach space X is uniformly non-square if and only if $f(X) > 2$ or $E(X) < 8$; (2) X has uniform normal structure provided $f(X) > 4(3 - \sqrt{5})$ or $E(X) < 3 + \sqrt{5}$.

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