

## MATRIX VERSIONS OF YOUNG'S INEQUALITY

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**Abstract.** A matrix majorization version of the general Young's inequality  $xy \leq \Phi(x) + \Psi(y)$  is presented. Multivariate Young's inequality is extended to the matrix setting by means of the geometric mean of positive semidefinite matrices. Also some refined Hilbert-Schmidt norm generalizations of Young's inequality are given and a symmetrized Young's inequality for unitarily invariant norms is proved.

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