

REVERSE INEQUALITIES OF ERDŐS–MORDELL TYPE

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Abstract. This paper deals with the reverse inequalities of Erdős–Mordell type. Our result contains as special case the following reverse Erdős–Mordell inequality:

$$R_1 + R_2 + R_3 < \sqrt{2}(\rho_1 + \rho_2 + \rho_3),$$

where R_i and ρ_i ($i=1, 2, 3$) denote respectively the distances from an interior point Q of $\triangle A_1A_2A_3$ to the vertexes A_1, A_2, A_3 and to the circumcenters of $\triangle A_2QA_3$, $\triangle A_3QA_1$, $\triangle A_1QA_2$. Some other closely related inequalities are also considered.

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