

COMPARISON OF LOCATION ESTIMATORS USING BANKS' CRITERION

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Abstract. In this paper, we analyze further Banks' (1997) closeness criterion for estimators, which is an alternative to Pitman's (1937) closeness criterion. We mainly concentrate our analysis on location estimation, and justify a conjecture by Banks (1997) that for heavy tail distributions the sample median is better than the sample mean when estimating a location parameter. The conclusion is reversed for distributions with lighter tails. To achieve this, we use asymptotics and exact probability calculations.

Mathematics subject classification (2000): 62F10, 62F12, 26D07, 26D15.

Keywords and phrases: Banks' criterion; Gautschi's inequality; location estimation; Pitman's criterion.

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