

## SOME NEW HARDY TYPE INEQUALITIES WITH GENERAL KERNELS

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*Abstract.* We state and prove some new weighted Hardy type inequalities with an integral operator  $A_k$  defined by

$$A_k f(x) := \frac{1}{K(x)} \int_{\Omega_2} k(x, y) f(y) d\mu_2(y),$$

where  $k : \Omega_1 \times \Omega_2 \rightarrow \mathbb{R}$  is a general nonnegative kernel,  $(\Omega_1, \mu_1)$  and  $(\Omega_2, \mu_2)$  are measure spaces and

$$K(x) := \int_{\Omega_2} k(x, y) d\mu_2(y), x \in \Omega_1.$$

In particular, the obtained results unify and generalize most of the results of this type (including the classical ones by Hardy, Hilbert and Godunova).

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