

## A SHARP CONVERSE INEQUALITY OF THREE WEIGHTED ARITHMETIC AND GEOMETRIC MEANS OF POSITIVE DEFINITE OPERATORS

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*Abstract.* In this paper we consider three-weighted arithmetic and geometric means of positive definite operators constructed by the symmetrization method that appeared in the definition of Ando-Li-Mathias's geometric mean of several positive definite operators and we establish a converse inequality of three-weighted arithmetic-geometric means via Specht ratio.

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