

DISCRETE MOMENT PROBLEMS WITH DISTRIBUTIONS KNOWN TO BE UNIMODAL

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Abstract. Discrete moment problems with given finite supports and unimodal distributions with known mode, are formulated and used to obtain sharp lower and upper bounds for expectations of higher order convex functions of discrete random variables as well as probabilities of the union of events. The bounds are based on the knowledge of some of the power moments of the random variables involved, or the binomial moments of the number of events which occur. The bounding problems are formulated as LP's and dual feasible basis structure theorems as well as the application of the dual method of linear programming provide us with the results. Applications in PERT and reliability are presented.

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