

A BIENAYMÉ–CHEBYSHEV INEQUALITY FOR SCALE MIXTURES OF THE MULTIVARIATE NORMAL DISTRIBUTION

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Abstract. In this short note a Chebyshev type sharp upper bound is presented for the tail probability of scale mixtures of the zero mean multivariate normal distribution, only in terms of the variance. Similar estimation is proved for the probability content of an arbitrary ellipsoid containing the origin.

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