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ON THE GODUNOVA–LEVIN–SCHUR CLASS OF FUNCTIONS

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Abstract. In 1985 Godunova and Levin have considered the following class of functions. A function $f: I \to \mathbb{R}$ is said to belong to the class $Q(I)$ if it is nonnegative and for all $x,y \in I$ and $t \in (0,1)$, satisfies the inequality:

$$f((1-t)x+ty) \leqslant \frac{f(x)}{1-t} + \frac{f(y)}{t}$$

Here I is an interval of $\mathbb{R}$.

It is known that all nonnegative quasiconvex functions belong to this class and this class of functions coincides with the class of Schur functions $\mathscr{S}(I)$, that is, with the class of nonnegative functions that satisfy the inequality

$$\sum f(x)(x-y)(x-z) \geqslant 0 \qquad \text{for every} \qquad x,y,z \in I$$

The aim of this paper is to survey some important properties of functions belonging to these classes of functions and to prove some new results concerning properties of functions from them.

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