

ALTERNATIVE CRITERIA FOR THE BOUNDEDNESS OF VOLTERRA INTEGRAL OPERATORS IN LEBESGUE SPACES

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Abstract. Three different criteria for $L_p - L_q$ boundedness of Volterra integral operator (??) with locally integrable weight functions w, v and a non-negative kernel $k(x, y)$ satisfying Oinarov's condition for each case $1 < p \leq q < \infty$ and $1 < q < p < \infty$ are given. Relations between components of the boundedness constants are described.

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