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AN EXTENSION OF THE FUGLEDE–PUTNAM'S THEOREM TO CLASS A OPERATORS

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Abstract. The familiar Fuglede-Putnam's Theorem is as follows (see [5], [9] and [11]): If A and B are normal operators and if X is an operator such that $AX = XB$, then $A^*X = XB^*$. In this paper, the hypothesis on A and B can be relaxed by using a Hilbert-Schmidt operator X: Let A be a class A operator and let B^* be an invertible class A operator such that $AX = XB$ for a Hilbert-Schmidt operator X. Then $A^*X = XB^*$. As a consequence of this result, we obtain that the range of the generalized derivation induced by this class of operators is orthogonal to its kernel. Some properties of log-hyponormal operators are also given.

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