

## SCHATTEN $p$ -NORM INEQUALITIES RELATED TO A CHARACTERIZATION OF INNER PRODUCT SPACES

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*Abstract.* Let  $A_1, \dots, A_n$  be operators acting on a separable complex Hilbert space such that  $\sum_{i=1}^n A_i = 0$ . It is shown that if  $A_1, \dots, A_n$  belong to a Schatten  $p$ -class, for some  $p > 0$ , then

$$2^{p/2} n^{p-1} \sum_{i=1}^n \|A_i\|_p^p \leq \sum_{i,j=1}^n \|A_i \pm A_j\|_p^p$$

for  $0 < p \leq 2$ , and the reverse inequality holds for  $2 \leq p < \infty$ . Moreover,

$$\sum_{i,j=1}^n \|A_i \pm A_j\|_p^2 \leq 2n^{2/p} \sum_{i=1}^n \|A_i\|_p^2$$

for  $0 < p \leq 2$ , and the reverse inequality holds for  $2 \leq p < \infty$ . These inequalities are related to a characterization of inner product spaces due to E.R. Lorch.

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