

APPROXIMATIONS TO EULER'S CONSTANT

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Abstract. We study a problem of finding good approximations to Euler's constant $\gamma = \lim_{n \rightarrow \infty} S_n$, where $S_n = \sum_{k=1}^n \frac{1}{k} - \log(n+1)$, by linear forms in logarithms and harmonic numbers. In 1995, C. Elsner showed that slow convergence of the sequence S_n can be significantly improved if S_n is replaced by linear combinations of S_n with integer coefficients. In this paper, considering more general linear transformations of the sequence S_n we establish new accelerating convergence formulae for γ . Our estimates sharpen and generalize recent Elsner's, Rivoal's and author's results.

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