

ON HILBERT'S INTEGRAL INEQUALITY AND ITS APPLICATIONS

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Abstract. In this paper it is shown that a new improvement on Hilbert's integral inequality can be established by introducing a weight function of the form $\left(\frac{1}{1+\sqrt{x}} - \frac{1}{1+x}\right)$ (with $x \geq 0$). As applications, some refinements on Widder's inequality and Hardy-Littlewood's inequality are given.

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