Mathematical
Inequalities
& Applications
Volume 14, Number 2 (2011), 343–357

WILLMORE LAGRANGIAN SUBMANIFOLDS IN COMPLEX PROJECTIVE SPACE

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Abstract. Let M be an n-dimensional compact Willmore Lagrangian submanifold in a complex projective space CP^n and let S and H be the squared norm of the second fundamental form and the mean curvature of M. Denote by $\rho^2 = S - nH^2$ the non-negative function on M, K and Q the functions which assign to each point of M the infimum of the sectional curvature and Ricci curvature at the point. We prove some integral inequalities of Simons' type for n-dimensional compact Willmore Lagrangian submanifolds in CP^n in terms of ρ^2, K, Q and H and obtain some characterization theorems.

Mathematics subject classification (2010): 53C42, 53C40.
Keywords and phrases: Willmore Lagrangian submanifolds; complex projective space; curvature; totally umbilical.

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Paper MIA-14-28

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