

SIMULTANEOUS AND CONVERSE APPROXIMATION THEOREMS IN WEIGHTED LEBESGUE SPACES

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Abstract. In this paper we deal with the simultaneous and converse approximation by trigonometric polynomials of the functions in the Lebesgue spaces with weights satisfying so called Muckenhoupt's A_p condition.

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