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# SHARP INEQUALITIES BETWEEN MEANS

YUMING CHU AND BOYONG LONG

*Abstract.* For $p \in \mathbb{R}$ the $p$-th power mean $M_p(a,b)$, arithmetic mean $A(a,b)$, geometric mean $G(a,b)$, and harmonic mean $H(a,b)$ of two positive numbers $a$ and $b$ are defined by

$$M_p(a,b) = \begin{cases} \left(\frac{a^p+b^p}{2}\right)^{1/p}, & p \neq 0, \\ \sqrt{ab}, & p = 0, \end{cases}$$

$A(a,b) = (a+b)/2$, $G(a,b) = \sqrt{ab}$, and $H(a,b) = 2ab/(a+b)$, respectively.

In this paper, we answer the questions: For $\alpha \in (0,1)$, what are the greatest values $p, r$ and $m$, and the least values $q, s$ and $n$, such that the inequalities $M_p(a,b) \leqslant A^\alpha(a,b)G^{1-\alpha}(a,b) \leqslant M_q(a,b), M_r(a,b) \leqslant G^\alpha(a,b)H^{1-\alpha}(a,b) \leqslant M_s(a,b)$ and $M_m(a,b) \leqslant A^\alpha(a,b)H^{1-\alpha}(a,b) \leqslant M_n(a,b)$ hold for all $a,b > 0$ ?

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## REFERENCES

[1] H. ALZER, *A power mean inequality for the gamma function*, Monatsh. Math., **131** (2000), 179–188.
[2] H. ALZER AND W. JANOUS, *Solution of problem 8**, Crux. Math., **13** (1987), 173–178.
[3] H. ALZER AND S. L. QIU, *Inequalities for means in two variables*, Arch. Math., **80** (2003), 201–215.
[4] J. BUKOR, J. TÓTH AND L. ZSILINSZKY, *The logarithmic mean and the power mean of positive numbers*, Octogon Math. Mag., **2** (1994), 19–24.
[5] P. S. BULLEN, D. S. MITRINOVIĆ, AND P. M. VASIĆ, *Means and Their Inequalities*, D. Reidel Publishing Co., Dordrecht, 1988.
[6] J. CHEN AND B. HU, *The identric mean and the power mean inequalities of Ky Fan type*, Facta Univ. Ser. Math. Inform., **4** (1989), 15–18.
[7] P. A. HÄSTÖ, *Optimal inequalities between Seiffert's mean and power means*, Math. Inequal. Appl., **7** (2004), 47–53.
[8] C. O. IMORU, *The power mean and the logarithmic mean*, Internat. J. Math. Math. Sci., **5** (1982), 337–343.
[9] W. JANOUS, *A note on generalized Heronian means*, Math. Inequal. Appl., **4** (2001), 369–375.
[10] T. P. LIN, *The power mean and the logarithmic mean*, Amer. Math. Monthly, **81** (1974), 879–883.
[11] J. E. PEČARIĆ, *Generalization of the power means and their inequalities*, J. Math. Anal. Appl., **161** (1991), 395–404.
[12] K. C. RICHARDS, *Sharp power mean bounds for the Gaussian hypergeometric function*, J. Math. Anal. Appl., **308** (2005), 303–313.
[13] C. D. TARNAVAS AND D. D. TARNAVAS, *An inequality for mixed power means*, Math. Inequal. Appl., **2** (1999), 175–181.
[14] W. L. WANG, J. J. WEN AND H. N. SHI, *Optimal inequalities involving power means*, Acta Math. Sinica, **47** (2004), 1053–1062 (in Chinese).
[15] S. H. WU, *Generalization and sharpness of the power means inequality and their applications*, J. Math. Anal. Appl., **312** (2005), 637–652.


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