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REVERSES OF ANDO'S INEQUALITY FOR POSITIVE LINEAR MAPS

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Abstract. Ando's inequality says that if A and B are positive operators on a Hilbert space H and Φ is a positive linear map, then for each $\alpha \in [0,1]$

$$\Phi(A \sharp_\alpha B) \leqslant \Phi(A) \sharp_\alpha \Phi(B)$$

where the α-geometric mean is defined by

$$A \sharp_\alpha B = A^{\frac{1}{2}} \left(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}\right)^{\alpha} A^{\frac{1}{2}}.$$

In this paper, we give simple proofs of reverse Ando's inequalities: If A and B are positive operators such that $mA \leqslant B \leqslant MA$ for some scalars $0 < m \leqslant M$, then for each $\alpha \in [0,1]$

$$\Phi(A) \sharp_\alpha \Phi(B) \leqslant \Phi(A \sharp_\alpha B) - C(m,M,\alpha)\Phi(A)$$

where the Kantorovich constant for the difference $C(m,M,\alpha)$ is defined by

$$C(m,M,\alpha) = (\alpha-1)\left(\frac{M^\alpha - m^\alpha}{\alpha(M-m)}\right)^{\frac{\alpha}{\alpha-1}} + \frac{Mm^\alpha - mM^\alpha}{M-m}$$

for any real number $\alpha \in \mathbb{R}$.

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REFERENCES

[1] T. ANDO, *Concavity of certain maps on positive definite matrices and applications to Hadamard products*, Linear Algebra Appl., **26** (1979), 203–241.
[2] T. ANDO, *Reverse Hölder inequality via A-G inequality*, a private note.
[3] J.-C. BOURIN, E.-Y. LEE, M. FUJII AND Y. SEO, *A matrix reverse Hölder inequality*, Linear Algebra Appl., **431** (2009), 2154–2159.
[4] M. FUJII, E.-Y. LEE AND Y. SEO, *A difference counterpart to a matrix Hölder inequality*, Linear Algebra Appl., **432** (2010), 2565–2571.
[5] T. FURUTA, J. MIĆIĆ, J.E. PEČARIĆ AND Y. SEO, *Mond-Pečarić Method in Operator Inequalities*, Monographs in Inequalities 1, Element, Zagreb, 2005.
[6] F. KUBO AND T. ANDO, *Means of positive linear operators*, Math. Ann., **246** (1980), 205–224.
[7] E.-Y. LEE, *A matrix reverse Cauchy-Schwarz inequality*, Linear Algebra Appl., **430** (2009), 805–810.
[8] J. MIĆIĆ, J. PEČARIĆ AND Y. SEO, *Complementary inequalities to inequalities of Jensen and Ando based on Mond-Pečarić method*, Linear Algebra Appl., **318** (2000), 87–107.

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