

## THE MULTIDIMENSIONAL REVERSE HARDY INEQUALITIES

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*Abstract.* In this paper we characterize the validity of the multidimensional reverse Hardy inequalities

$$\|gw\|_{L_p(\mathbb{R}^n)} \leq C \left\| v(t) \int_{B(0,t)} g(y) dy \right\|_{L_q(0,+\infty)}$$

and

$$\|gw\|_{L_p(\mathbb{R}^n)} \leq C \left\| v(t) \int_{B(0,t)} g(y) dy \right\|_{L_q(0,+\infty)}$$

for non-negative measurable functions on  $\mathbb{R}^n$ , where  $B(0,t)$  is the closed ball in  $\mathbb{R}^n$  centered at zero with radius  $t$ ,  ${}^c B(0,t) = \mathbb{R}^n \setminus B(0,t)$ ,  $0 < p \leq 1$ ,  $0 < q \leq +\infty$ ,  $w$  and  $v$  are weight functions on  $\mathbb{R}^n$  and  $(0, +\infty)$ , respectively. Obtained conditions are the natural extensions of one-dimensional conditions.

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