

TWO SHARP INEQUALITIES FOR TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS

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Abstract. We determine the best positive constants p and q such that

$$(\sinh x/x)^p < x/\sin x < (\sinh x/x)^q.$$

Some applications for Wilker's type inequalities are given.

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