Mathematical
Inequalities
& Applications
Volume 15, Number 3 (2012), 571–580 doi:10.7153/mia-15-50

QUADRATIC INEQUALITIES AND A CHARACTERIZATION OF INNER PRODUCT

JANUSZ MATKOWSKI

Abstract. Let X be a real linear space and let $a \in \mathbb{R}$, $a \neq 0$, be fixed. Assuming that the functions $g,h : X \to \mathbb{R}$ satisfy the inequalities $g(ax+y)+h(x-ay) \leqslant a^2 g(x)+g(y)+h(x)+a^2 h(y)$ for all $x,y \in X$, and some subhomogeneity type conditions, we prove that $h = g$, the function g is a quadratic functional, and there exists a unique symmetric biadditive function $S : X^2 \to \mathbb{R}$ such that $g(x) = S(x,x)$ for all $x \in X$.

A motivation in the theory of orthogonal additive functions is presented.

Mathematics subject classification (2010): Primary 46C15, 26D15, 39B22; Secondary 39B55.

Keywords and phrases: parallelogram equality, inner product, quadratic functional, quadratic inequality, Jordan-von Neumann theorem.

REFERENCES

[1] J. ACZÉL & J. DHOMBRES, *Functional equations in several variables*, Encyclopedia of Mathematics and its Applications, Cambridge University Press, Cambridge-New York-New Rochelle-Melbourne-Sydney, 1989.

[2] D. AMIR, *Characterization of inner product spaces*, Birkhäuser Verlag, Basel-Boston-Stuttgart, 1986.

[3] M. M. DAY, *Some characterizations of inner product spaces*, Trans. Amer. Math. Soc. **62** (1947), 320–337.

[4] P. JORDAN, J. VON NEUMANN, *On inner products in linear metric spaces,* Ann. Math. **16** (1935), 719–723.

[5] L. SZÉKELYHIDI, *Convolution Type Functional Equations on Topological Abelian Groups*, World Scientific Publishing Co. Inc., Teaneck, NJ, 1991.