

## TWO GENERALIZATIONS OF LANDAU'S INEQUALITY

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*Abstract.* Let  $\pi(x)$  be the number of prime numbers not exceeding  $x$ . In the introduction we present some inequalities related to this function; these have been presented in several articles about the *Number Theory*. In the present paper we obtain two inequalities which generalize Landau's Inequality,  $\pi(2x) \leq 2\pi(x)$  for any integer  $x \geq 2$ . Also, we obtain the inequality  $2[x\pi(x) + y\pi(y)] > (x+y)\pi(x+y)$ , for all integers  $x, y \geq 67$ , and an inequality which refers to the Hardy and Littlewood conjecture. To demonstrate them, we used the Personal Computer, in order to extend the domain of the variables for which these inequalities are true.

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