

SOME MONOTONICITY PROPERTIES OF GENERALIZED ELLIPTIC INTEGRALS WITH APPLICATIONS

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Abstract. In this paper, we present some monotonicity theorems involving the generalized elliptic integrals $\mathcal{K}_a(r)$ and $\mathcal{E}_a(r)$, and find an asymptotic property of $\mathcal{K}_a(r)$ when $r \rightarrow 1$. As applications, some well known results are improved.

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