

APPLICATIONS OF FAN'S MATCHING THEOREM IN MKKM SPACES

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Abstract. Using some concepts in MKKM space, we prove the generalized forms of open and closed versions of Fan's matching theorem. The S -weakly KKM multimap is applied to obtain a type of KKM theorem. As an application, a Fan type minimax theorem is proved. Moreover, a minimax inequality is given in these new settings.

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