

THE ORLICZ AFFINE ISOPERIMETRIC INEQUALITY

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Abstract. In this paper, the Orlicz-affine surface area is introduced. Isoperimetric inequalities for this new affine surface area are established.

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REFERENCES

- [1] A. ALESKER, *Continuous rotation invariant valuation on convex sets*, Ann. of Math. **149** (1999), 977–1005.
- [2] B. ANDREWS, *Contraction of convex hypersurfaces by their affine normal*, J. Differential Geom. **43** (1996), 207–236.
- [3] B. ANDREWS, *The affine curve-lengthening flow*, J. Reine Angew. Math. **506** (1999), 43–83.
- [4] W. BLASCHKE, *Differential geometry II*, Springer, Berlin, 1923.
- [5] F. CHEN, J. ZHOU, AND C. YANG, *On the reverse Orlicz Busemann-Petty centroid inequality*, Adv. Appl. Math. **47** (2011), 820–828.
- [6] G. EWALD, D. LARMAN, AND C. ROGERS, *The directions of the line segments and of the r -dimensional balls on the boundary of a convex body in Euclidean space*, Mathematika **17** (1970), 1–20.
- [7] P. GRUBER, *Aspects of approximation of convex bodies*, Handbook of Convex Geometry **A** (1993), 321–345.
- [8] C. HABERL, E. LUTWAK, D. YANG, AND G. ZHANG, *The even Orlicz Minkowski problem*, Adv. Math. **224** (2010), 2485–2510.
- [9] D. HUG, *Contributions to affine surface area*, Manuscripta Math. **91**, No. 3 (1996), 283–301.
- [10] K. LEICHTWEISS, *Über ein Formel Blaschkes zur affinoberfläche*, Studia Sci. Math. Hungar **21** (1986), 453–474.
- [11] M. LUDWIG, *General affine surface areas*, Adv. Math. **224** (2010), 2346–2360.
- [12] M. LUDWIG AND M. REITZNER, *A characterization of affine surface area*, Adv. Math. **147** (1999), 138–172.
- [13] M. LUDWIG AND M. REITZNER, *A classification of $SL(n)$ invariant valuations*, Ann. of Math. **172** (2010), 1223–1271.
- [14] M. LUDWIG, C. SCHÜTT, AND E. WERNER, *Approximation of the Euclidean ball by polytopes*, Studia Math. **173** (2006), 1–18.
- [15] E. LUTWAK, *On some affine isoperimetric inequalities*, J. Differential Geo. **23** (1986), 1–13.
- [16] E. LUTWAK, *Centroid bodies and dual mixed volumes*, Proc. London Math. Soc. **60** (1990), 365–391.
- [17] E. LUTWAK, *Extended affine surface area*, Adv. Math. **85** (1991), 39–68.
- [18] E. LUTWAK, *The Brunn-Minkowski-Firey Theory I: Mixed volumes and the Minkowski problem*, J. Differential Geom. **38** (1993), 131–150.
- [19] E. LUTWAK, *The Brunn-Minkowski-Firey Theory II: Affine and geominimal surface area*, Adv. Math. **118** (1996), 224–194.
- [20] E. LUTWAK AND V. OLIKER, *On the regularity of the solution of a generalization of the Minkowski problem*, J. Differential Geom. **41** (1995), 227–246.
- [21] E. LUTWAK, D. YANG, AND G. ZHANG, *Orlicz centroid bodies*, J. Differential Geo. **84** (2010), 365–387.

- [22] E. LUTWAK, D. YANG, AND G. ZHANG, *Orlicz projection bodies*, Adv. Math. **223** (2010), 220–242.
- [23] M. MEYER AND A. PAJOR, *On the Blaschke Santaló inequality*, Arch. Math. **55** (1990), 82–93.
- [24] M. MEYER AND E. WERNER, *On the p -affine surface area*, Adv. Math. **152** (2000), 288–313.
- [25] G. PAOURIS AND E. WERNER, *Relative entropy of cone measures and L_p centroid bodies*, Proc. Landon Math. Soc. **104** (2012), 253–286.
- [26] C. PETTY, *Geominimal surface area*, Geom. Dedicata **3** (1974), 77–97.
- [27] C. PETTY, *Affine isoperimetric problem*, Ann. N. Y. Acad. Sci. **440** (1985), 113–127.
- [28] S. REISNER, C. SCHÜTT, AND E. WERNER, *A note on Mahler's conjecture*, Int. Math. Res. Notices (2011), DOI: 10.1093/imrn/rnr003.
- [29] M. SCHMUCKENSHLÄGER, *The distribution function of the convolution square of a convex symmetric body in R^n* , Israel J. Math. **78** (1992), 309–334.
- [30] R. SCHNEIDER, *Convex bodies: The Brunn-Minkowski Theory*, Cambridge University Press, Cambridge, 1993.
- [31] C. SCHÜTT, *On the affine surface area*, Proc. Amer. Math. Soc. **118** (1993), 1213–1218.
- [32] C. SCHÜTT, *Random polytopes and affine surface area*, Math. Nachr. **170** (1994), 227–249.
- [33] C. SCHÜTT AND E. WERNER, *The convex floating body*, Math. Scand. **66** (1990), 275–290.
- [34] C. SCHÜTT AND E. WERNER, *Sufface bodies and p -affine surface area*, Adv. Math. **187** (2004), 98–145.
- [35] N. TRUDINGER AND X. WANG, *The affine plateau problem*, J. Amer. Math. Soc. **18** (2005), 253–289.
- [36] X. WANG, *Affine maximal hypersurfaces*, Proc. The International Congress of Mathematicians **3** (2002), 221–231.
- [37] E. WERNER, *Illumination bodies and affine surface area*, Studia Math. **110** (1994), 257–269.
- [38] E. WERNER, *On L_p affine surface areas*, Indiana Univ. Math. **56** (2007), 2305–2324.
- [39] E. WERNER, *Renyi Divergence and L_p affine surface area for convex bodies*, Adv. Math. **230** (2012), 1040–1059.
- [40] E. WERNER AND D. YE, *New L_p affine isoperimetric inequalities*, Adv. Math. **218** (2008), 762–780.
- [41] E. WERNER AND D. YE, *Inequalities for mixed p -affine surface area*, Math. Ann. **347** (2010), 703–737.
- [42] D. YE, *Inequalities for general mixed affine surface area*, J. London Math. Soc. **85** (2012), 101–120.