

ON THE FUNCTIONAL BLASCHKE–SANTALÓ INEQUALITY

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Abstract. In this paper, using functional Steiner symmetrizations, we show that Meyer and Pajor’s proof of the Blaschke-Santaló inequality can be extended to the functional setting.

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