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# PRODUCTS OF NONCOMMUTATIVE CALDERÓN–LOZANOVSKIĬ SPACES

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*Abstract.* Let $\mathscr{M}$ be a semifinite von Neumann algebra with a normal semifinite faithful trace $\tau$. We show that the noncommutative Calderón-Lozanovskiĭ spaces $E_\varphi(\mathscr{M})$ can be written in the form $E_\varphi(\mathscr{M}) = E_{\varphi_2}(\mathscr{M}) \cdot E_{\varphi_1}(\mathscr{M})$, if at least one of the following conditions holds:

(i) $\varphi_1^{-1}\varphi_2^{-1} \approx \varphi^{-1}$ for all arguments,

(ii) $\varphi_1^{-1}\varphi_2^{-1} \approx \varphi^{-1}$ for large arguments and $\mathscr{M} \hookrightarrow E(\mathscr{M})$,

(iii) $\varphi_1^{-1}\varphi_2^{-1} \approx \varphi^{-1}$ for small arguments and $E(\mathscr{M}) \hookrightarrow \mathscr{M}$.

Here $E_{\varphi_2}(\mathscr{M}) \cdot E_{\varphi_1}(\mathscr{M})$ denote the product of the noncommutative Calderón-Lozanovskiĭ spaces $E_{\varphi_1}(\mathscr{M})$ and $E_{\varphi_2}(\mathscr{M})$.

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