

## THE HERMITIAN AND NONNEGATIVE DEFINITE SOLUTIONS OF $AX = B$ SUBJECT TO $CXC^* \geq D$

XIFU LIU

*Abstract.* In this paper, we first establish some necessary and sufficient conditions for the existence of Hermitian and nonnegative definite solutions of  $AX = B$  subject to  $CXC^* \geq D$ , where  $D$  is Hermitian matrix. Furthermore, general expressions for this constrained Hermitian and nonnegative definite solutions are derived, several special cases are also considered.

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