

HARTLEY–FOURIER COSINE GENERALIZED CONVOLUTION INEQUALITIES

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Abstract. In this paper, we study some inequalities related to a certain generalized convolution for the Hartley-Fourier cosine integral transforms. Specially, we will apply these inequalities to estimate the solutions of some integral equations, differential equations and partial differential equations.

Mathematics subject classification (2010): 44A35, 45E10, 42A38.

Keywords and phrases: Convolution, Hartley transform, Fourier transform, Fourier cosine transform, convolutions inequalities.

REFERENCES

- [1] M. ABRAMOWITZ AND I. A. STEGUN, *Handbook of Mathematical Functions, with Formulas, Graphs and Mathematical Tables*, National Bureau of Standards Applied Mathematics Series, **55**, Washington, D.C., 1964.
- [2] R. A. ADAMS AND J. J. F. FOURNIER, *Sobolev Spaces, 2nd ed.*, Academic Press, New York, Amsterdam, Elsevier Science, 2003.
- [3] R. N. BRACEWELL, *The Hartley transform*, Oxford University Press, Clarendon Press, New York, 1986.
- [4] M. ABRAMOWITZ AND I. A. STEGUN, A. ERDELYI ET AL., *Table of Integral Transforms*, Vol. I. McGraw-Hill Book Co., New York-Toronto-London, 1954.
- [5] N. T. HONG, *Fourier cosine convolution inequalities and applications*, Integral Transforms Special Functions, **10**, 21 (2010), 755–763.
- [6] H. H. KAGIWADA AND R. KALABA, *Integral Equations via Imbedding Methods*, Applied Mathematics and Computation, No. 6. Addison-Wesley Publishing Co., Reading-Mass.-London-Amsterdam, 1974.
- [7] D. S. MITRINOVIĆ, J. E. PEČARIĆ, AND A. M. FINK, *Classical and New inequalities in Analysis*, Kluwer Academic Published, The Netherlands, 1993.
- [8] I. N. SNEDDON, *The Use of Integral Transforms*, McGraw-Hill. New York, 1972.
- [9] S. SAITOH, *A fundamental inequality in the convolution of L_2 functions on the half line*, Proc. Amer. Math. Soc., **91** (1984), 285–286.
- [10] S. SAITOH, *Inequalities in the most simple Sobolev space and convolutions of L_2 functions with weights*, Proc. Amer. Math. Soc., **118** (1993), 515–520.
- [11] V. K. TUAN, AND M. YAMAMOTO, *Convolution inequalities and applications*, Journal of Inequalities in Pure and Applied Mathematics, **3**, 4 (2003), Article 50.
- [12] S. SAITOH, V. K. TUAN, AND M. YAMAMOTO, *Reverse weighted L_p -norm inequalities in convolutions and stability in inverse problems*, Journal of Inequalities in Pure and Applied Mathematics, **1**, 1 (2000), Article 7. [Online: <http://jipam.vu.edu.au/v1n1/018-99.html>].
- [13] N. X. THAO, V. K. TUAN, AND H. T. V. ANH, *On the Toeplitz plus Hankel integral equation II*, Integral Transforms and Special Functions, **1**, 25 (2014), 75–84.
- [14] E. C. TITCHMARCH, *Introduction to the Theory of Fourier integrals, 3rd Ed.* Chelsea publishing Co., New York, 1986.

- [15] J. N. TSITSIKLIS, AND B. C. LEVY, *Integral equations and resolvents of Toeplitz plus Hankel kernels*, Laboratory for Information and Decision Systems, Massachusetts Institute of Technology. Series/Report No.: LIDS-P 1170, 1981.
- [16] VU KIM TUAN, S. B. YAKUBOVICH, *On the criterion of unitary of the bilateral integral transform*, *Ukranian Math. J.* **5**, 44 (1992), 697–700 (In Russian). English transl.: *A criterion for a two-sided integral transform to be unitary*, *Ukr. Math J.* **5**, 44 (1992), 630–632.