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ON CRITICAL CONDITION FOR A WEIGHTED INTEGRAL SYSTEM WITH NEGATIVE EXPONENTS

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Abstract. This paper is concerned with the integral system

$$\begin{cases} u(x) = \int_{R^n} |x|^\alpha |y|^\beta |x-y|^s v^q(y)dy, & u > 0 \text{ in } R^n, \\ v(x) = \int_{R^n} |x|^\beta |y|^\alpha |x-y|^s u^p(y)dy, & v > 0 \text{ in } R^n, \end{cases}$$

where $n \geqslant 1$, $\alpha,\beta,s > 0$ and $p,q < 0$. Such an integral system appears in the study of the conformal geometry and the weighted Hardy-Littlewood-Sobolev inequality. We obtain that

$$\frac{1}{p+1} + \frac{1}{q+1} = -\frac{\alpha+\beta+s}{n},$$

is a necessary condition for the existence of the C^1 positive entire solutions, which is also the necessary and sufficient condition for the invariant of the system and some energy functionals under the scaling transformation.

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