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ASYMPTOTIC EXPANSIONS OF INTEGRAL MEAN OF POLYGAMMA FUNCTIONS

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Abstract. Let s,t be two given real numbers, $s \neq t$ and $m \in \mathbb{N}$. We determine the coefficients $a_j(s,t)$ in the asymptotic expansion of integral (or differential) mean of polygamma functions $\psi^{(m)}(x)$:

$$\frac{1}{t-s}\int_s^t \psi^{(m)}(x+u)\,\mathrm{d}u \sim \psi^{(m)}\left(x\sum_{j=0}^{\infty}\frac{a_j(s,t)}{x^j}\right), \quad x\to\infty.$$

We derive the recursive relations for polynomials $a_j(t,s)$, and also as polynomials in intrinsic variables $\alpha = \frac{1}{2}(s+t-1)$, $\beta = \frac{1}{4}[1-(t-s)^2]$. We derive also the main properties of these polynomials and as a consequence the asymptotic formula for shifted variables.

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