

LYAPUNOV–TYPE INEQUALITIES FOR A FRACTIONAL DIFFERENTIAL EQUATION WITH MIXED BOUNDARY CONDITIONS

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Abstract. Lyapunov-type inequalities are established for a fractional differential equation under mixed boundary conditions. Using such inequalities, we obtain intervals where certain Mittag-Leffler functions have no real zeros.

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