

## RADIAL $P$ -TH MOMENT OF A RANDOM VECTOR

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*Abstract.* In this paper, we first introduce a new concept – radial  $p$ -th moment of a random vector for star body, which is a general form of the standard  $p$ -th moment. Further we establish some properties of the radial  $p$ -th moment and give some related applications.

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### REFERENCES

- [1] Z. ARTSTEIN AND R. A. VITALE, *A strong law of large numbers for random compact sets*, Ann. Probab. **5** (3) (1975), 879–882.
- [2] J. BOURGAIN AND V. D. MILMAN, *New volume ratio properties for convex symmetric bodies in  $R^n$* , Invent. Math. (**88**) (1987), 319–340.
- [3] K. M. BALL, *An elementary introduction to modern convex geometry*, Flavors of Geometry, ed. by Silvio Levy, Cambridge University Press, New York, (**31**) (1997), 1–58.
- [4] M. H. M. COSTA AND T. M. COVER, *On the similarity of the entropy power inequality and the Brunn-Minkowski inequality*, IEEE Trans. Inform. Theory. **6** (IT-30) (1984), 837–839.
- [5] T. M. COVER, A. DEMBO, AND J. A. THOMAS, *Information theoretic inequalities*, IEEE Trans. Inform. Theory. (**37**) (1991), 1501–1518.
- [6] T. M. COVER AND J. A. THOMAS, *Elements of information theory*, New York: Wiley-Interscience, (1991).
- [7] D. CORDERO-ERAUSQUIN, B. NAZARET, AND C. VILLANI, *A mass-transportation approach to sharp Sobolev and Gagliardo-Nirenberg inequalities*, Adv. Math. **2** (182) (2004), 307–332.
- [8] A. DEMBO, *Information inequalities and uncertainty principles*, Dep. Statist., Stanford Univ., Stanford, CA, Tech. Rep. (1991).
- [9] R. J. GARDNER, *Geometric tomography*, Cambridge University Press, Cambridge, (1995).
- [10] P. M. GRUBER, *Convex and discrete geometry*, Springer-Verlag, New York, (2007).
- [11] O. G. GULERYUZ, E. LUTWAK, D. YANG, AND G. ZHANG, *Information-theoretic inequalities for contoured probability distributions*, IEEE Trans. Inform. Theory. **8** (48) (2002), 2377–2383.
- [12] R. HE AND G. LENG, *A strong law of large numbers on the harmonic  $p$ -combination*, Geom. Dedicata, (**154**) (2011), 103–116.
- [13] R. HE AND G. LENG, *A probabilistic version of quermassintegrals’ Brunn-Minkowski inequality* (in Chinese), Sci. Sin. Math, (**43**) (2013), 941–952.
- [14] B. S. KASIN, *The widths of certain finite-dimensional sets and classes of smooth functions*, Izv. Akad. Nauk SSSR Ser. Mat. **2** (41) (1977), 334–351.
- [15] E. LUTWAK, *The Brunn-Minkowski-Firey theory II: Affine and Geominimal Surface Areas*, Adv. Math. (**118**) (1996), 244–294.
- [16] E. LUTWAK, D. YANG, AND G. ZHANG, *A new ellipsoid associated with convex bodies*, Duke Math. J. **3** (104) (2000), 375–390.
- [17] E. LUTWAK, D. YANG, AND G. ZHANG,  *$L_p$  affine isoperimetric inequalities*, J. Differential Geom. **1** (56) (2000), 111–132.
- [18] E. LUTWAK, D. YANG, AND G. ZHANG, *The Cramer-Rao inequality for star bodies*, Duke Math. J. **1** (112) (2002), 59–81.

- [19] E. LUTWAK, D. YANG, AND G. ZHANG, *Moment-entropy inequalities*, Ann. Probab. **(32)** (2004), 757–774.
- [20] E. LUTWAK, D. YANG, AND G. ZHANG, *Cramer-Rao and moment-entropy inequalities for Renyi entropy and generalized Fisher information*, IEEE Trans. Inform. Theory. **(51)** (2005), 473–478.
- [21] E. LUTWAK, D. YANG, AND G. ZHANG, *Moment-entropy inequalities for a random vector*, IEEE Trans. Inform. Theory. **(53)** (2007), 1603–1607.
- [22] E. LUTWAK, S. LV, D. YANG, AND G. ZHANG, *Extensions of Fisher Information and Stam's Inequality*, IEEE Trans. Inform. Theory. **(58)** (2012), 1319–1327.
- [23] E. LUTWAK, S. LV, D. YANG, AND G. ZHANG, *Affine moments of a random vector*, IEEE Trans. Inform. Theory. **(59)** (2013), 5592–5599.
- [24] S. LV, AND X. LV, *Affine Fisher information inequalities*, J. Math. Anal. Appl. **(371)** (2010), 347–354.
- [25] R. SCHNEIDER, *Convex bodies: the Brunn-Minkowski theory*, Cambridge University Press, Cambridge, 1993.
- [26] R. A. VITALE, *On the volume of parallel bodies: a probabilistic derivation of the Steiner formula*, Adv. in appl. Probab. **(27)** (1995), 97–101.
- [27] R. A. VITALE, *The Wills functional and Gaussian processes*, Ann. Probab. **(24)** (1996), 2172–2178.