

## A WEIGHTED HARDY-TYPE INEQUALITY FOR $0 < p < 1$ WITH SHARP CONSTANT

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*Abstract.* Hardy-type inequalities with sharp constants for  $0 < p < 1$  for power weight functions were established in [10], [5]. In this work, we give an extension of these inequalities for general weight functions, prove the existence of extremal functions and write them out explicitly.

*Mathematics subject classification (2010):* 35J20, 35J25.

*Keywords and phrases:* Hardy operator, weighted  $L_p$ -spaces with  $0 < p < 1$ .

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