

## CHARACTERIZATIONS OF INNER PRODUCT SPACES BY INEQUALITIES INVOLVING SEMI-INNER PRODUCT

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*Abstract.* Using the notion of semi-inner product in normed spaces, in this paper we provide some new characterizations of inner product spaces. We answer a question posed by Dragomir, whether the property (N) is characteristic for inner product spaces. We show that the space  $X$  is of (N)-type if and only if the norm in  $X$  comes from an inner product.

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