

NUMERICAL RADIUS INEQUALITIES ASSOCIATED WITH THE CARTESIAN DECOMPOSITION

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Abstract. We give several sharp numerical radius inequalities associated with the Cartesian decomposition of a Hilbert space operator $A = B + iC$. Among other inequalities, it is shown that

$$\frac{1}{2} \| |B|^r + |C|^r \| \leq w^r(A) \leq \| |B|^r + |C|^r \|$$

for $0 < r \leq 2$, where $w(\cdot)$ and $\|\cdot\|$ denote the numerical radius and the usual operator norm, respectively. These inequalities generalize and extend earlier numerical radius inequalities.

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