

JESSEN'S FUNCTIONAL AND MAJORIZATION

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Abstract. In this note, we prove a Sherman type inequality for the Jessen's functional by using a majorization method. In consequence, we obtain a Hardy-Littlewood-Pólya-Karamata type inequality, which says that some n -sums generated by the Jessen's functional are Schur-concave with respect to its n -tuple of weight vectors.

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