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ADDITIVE FUNCTIONS AND THEIR ACTIONS ON CERTAIN ELEMENTARY FUNCTIONS

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Abstract. The main aim of this note is to provide sufficient conditions for an additive function to be a real derivation. Among others the following implication will be verified: Assume that $\xi\colon \mathbb{R} \to \mathbb{R}$ is a given differentiable function and for the additive function $d\colon \mathbb{R} \to \mathbb{R}$, the mapping

$$x \longmapsto d(\xi(x)) - \xi'(x)d(x)$$

is regular (e. g. measurable, continuous, locally bounded). Then d is a sum of a derivation and a linear function.

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