

## HARDY'S TYPE INEQUALITY FOR THE OVER CRITICAL EXPONENT ASSOCIATED WITH THE DUNKL TRANSFORM

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*Abstract.* For the Hardy space  $H_{q,s}^p(\mathbb{R}^d, \mu_k)$ ,  $0 < p \leq 1$ , we shall improve a Hardy's type inequality associated with the Dunkl transform respect to the measures  $\mu_\kappa$  homogeneous of degree  $\gamma$ , on the strip  $(2\gamma + d)(2 - p) \leq \sigma < 2\gamma + d + p(N + 1)$ , where  $N = [(2\gamma + d)(1/p - 1)]$  is the greatest integer not exceeding  $(2\gamma + d)(1/p - 1)$ .

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