

COMMUTATORS OF RIESZ TRANSFORMS WITH LIPSCHITZ FUNCTIONS RELATED TO MAGNETIC SCHRÖDINGER OPERATORS

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Abstract. Let $A := -(\nabla - i\vec{a}) \cdot (\nabla - i\vec{a}) + V$ be a magnetic Schrödinger operator on $L^2(\mathbb{R}^n)$, $n \geq 2$, where $\vec{a} := (a_1, \dots, a_n) \in L^2_{\text{loc}}(\mathbb{R}^n, \mathbb{R}^n)$ and $0 \leq V \in L^1_{\text{loc}}(\mathbb{R}^n)$. In this paper, the author shows that the commutators of the Riesz transforms $L_k A^{-1/2}$, $k \in \{1, \dots, n\}$, with functions in Lipschitz space $\text{Lip}_\alpha(\mathbb{R}^n)$ for $\alpha \in (0, 1)$, are bounded from $L^p(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)$, where $1/p - 1/q = \alpha/n$ and L_k is the closure of $\frac{\partial}{\partial x_k} - ia_k$ in $L^2(\mathbb{R}^n)$. Let ρ be an admissible function modeled on the known auxiliary function determined by the Schrödinger operator $-\Delta + V$. The author also characterizes a localized Lipschitz space $\text{Lip}_{\alpha, \rho}(\mathbb{R}^n)$ in terms of the localized Riesz transforms $\{\tilde{R}_j\}_{j=1}^n$ and their adjoint operators.

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