

## SOME SHARP INEQUALITIES FOR MULTIDIMENSIONAL INTEGRAL OPERATORS WITH HOMOGENEOUS KERNEL: AN OVERVIEW AND NEW RESULTS

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*Abstract.* One goal of this paper is to point out the fact that a big number of inequalities proved from time to time in journal publications, both one-dimensional and multi-dimensional, are particular cases of some general results for integral operators with homogeneous kernels, including in particular, the statements on sharp constants.

Some new multidimensional Hardy-Hilbert type inequalities are derived. Moreover, a new multidimensional Pólya-Knopp inequality is proved and some examples of applications are derived from this result. The constants in all inequalities are sharp.

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